



5th International Symposium on Master Engineering *Booklets*



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Title: Mathematical processing of multi- and hyperspectral images for engineering applications

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Editorial label MARVID: 607-8695
BMARVID Control Number: 2025-01
BMARVID Classification (2025): 021025-0001

Pages: 32

RNA: 03-2010-032610115700-14

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Maestría en Ciencia en Ingeniería Eléctrica

El programa de Maestría en Ciencias del Cinvestav, Unidad Guadalajara, tiene como finalidad la formación de recursos humanos capaces de utilizar el método científico para resolver problemas científicos, tecnológicos o de ingeniería, así como ejercer la docencia a nivel superior y de postgrado.

Duración del programa

El programa tiene una duración de 2 años, siendo el tiempo mínimo 1.5 años.

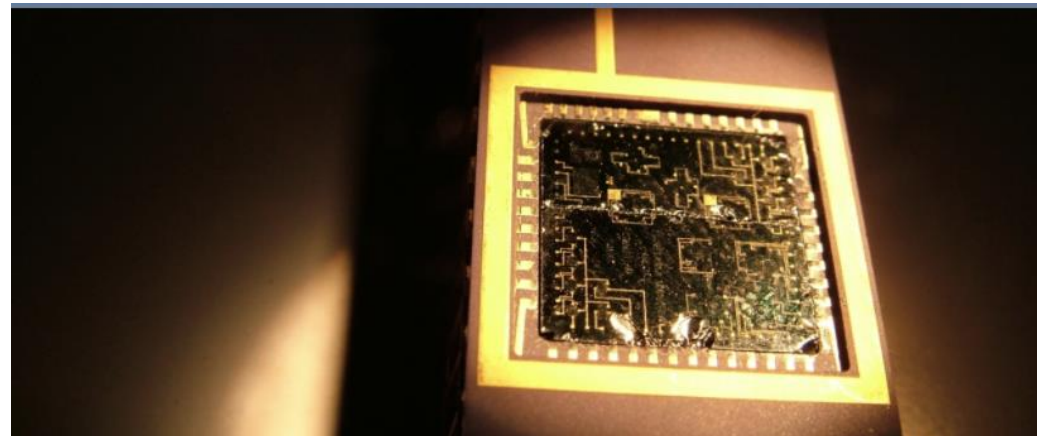
El programa es de tiempo completo, no tenemos la modalidad de tiempo parcial.

Costo

La maestría en ciencias no tiene ningún costo para el alumno. Los alumnos aceptados serán postulados para la beca de Conahcyt (información acerca de la beca)

Líneas de Intensificación

- Ciencias de la Computación.
- Control Automático.
- Diseño Electrónico.
- Sistemas Eléctricos y Potencia.
- Telecomunicaciones.



Contenido: YouTubeTelecomCinvesGDL

- 1) Introduction
- 2) State of the Art: The most important methods, techniques, and algorithms
- 3) Motivation: Unsolved Communication Problems
- 4) Problem Statements and Mathematical Models that define the Problems, Objectives
- 5) Problems:
 - 1) Background and Foreground Formation of Vehicle Traffic
 - 2) Moving Objects Detection and Tracking in a Video, ROI determination
 - 3) Wildfire Detection and Analysis
 - 4) Remote Sensing Problems

Some Elements of the Mathematical Basis

- 1) Linear and Multilinear Algebra or Tensor Algebra
- 2) Machine Learning: Feature Extraction and Input Data Space, Classifiers, Loss Functions, Output Data Space
- 3) Optimization and Regularization
- 4) Hadamard formulated three essential criteria for a mathematical problem to be well-posed:
 1. **Existence:** A solution exists.
 2. **Uniqueness:** The solution is unique.
 3. **Stability:** The solution depends continuously on the input data



Some Problems

A **high performance** vision-based system with one static camera for traffic surveillance, for moving vehicle detection with occlusion handling, tracking, counting, and One Class Support Vector Machine (OC-SVM) classification. Multiviews and Multitasks are present.



Lane detection for traffic surveillance in intelligent transportation systems is a challenge for vision-based systems. In this paper, a novel **pixel-entropy** based algorithm for the automatic detection of the number of lanes and their centers, as well as the **formation of their division lines** is proposed.

This work aims at addressing two issues simultaneously: **data compression** at input space and semantic segmentation. **Semantic segmentation** of remotely sensed **multi-** or **hyperspectral images** through deep learning (DL) artificial neural networks (ANN) delivers as output the corresponding matrix of pixels classified elementwise, achieving competitive performance metrics.

Representation Learning, process by learning transformations of raw data of the Input Data Space into representations or vectors, matrices and tensors with a lower dimensions. Then it makes easier the extraction of useful information.

Problems of Remote Sensing: it is the science for monitoring and analyzing Earth's surface and atmosphere. Applications: **Agriculture, Environmental Monitoring, Urban Planning and Development, Forestry, Climate and Weather, Military and Defense, and others.**



Content

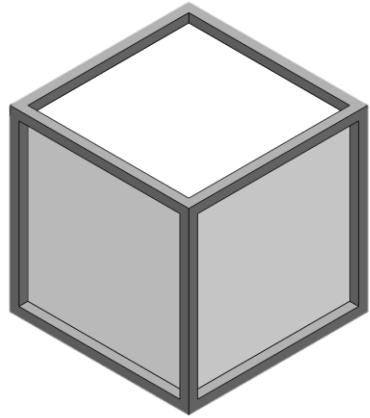
- V2 <https://youtu.be/SeEhrogzXec>
- V3 https://youtu.be/BDGyB7XDV_E
- V4 <https://youtu.be/L8rEPmMO4x4>
- V5 <https://youtu.be/csdjk7hhtcE>
- V6 <https://youtu.be/FMLTJGI0s1w>

Introduction: Physical-Mathematical Relationships



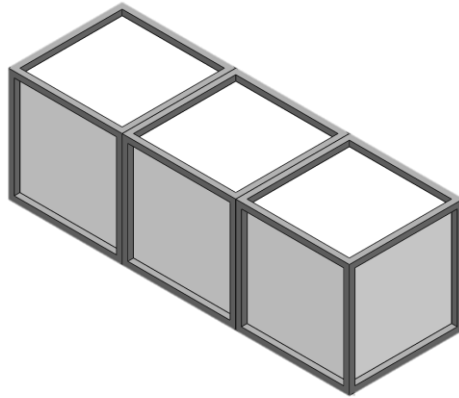
- Features or attributes, measurements, observations: **scalars**
- Particular set of features: **vectors**
- Moving vehicles: **vectors(t)**
- Images, pixels: **Matrices, elements or entries**
- Multispectral and Hyperspectral images: **Tensors or Hypermatrices**
- Videos: **Tensors**

Tensor Examples



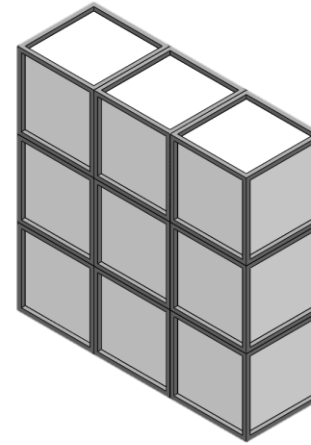
x : a scalar.

$$x \in \mathbb{R}$$



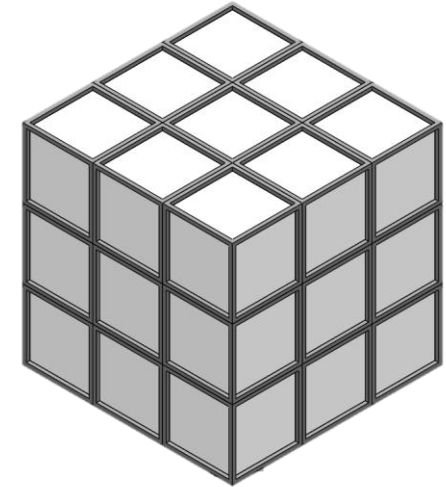
\mathbf{x} : a 1st-order tensor.

$$\mathbf{x} \in \mathbb{R}^3$$



\mathbf{X} : a 2nd-order tensor.

$$\mathbf{X} \in \mathbb{R}^{3 \times 3}$$



\mathcal{X} : a 3rd-order tensor.

$$\mathcal{X} \in \mathbb{R}^{3 \times 3 \times 3}$$

Tensor Examples

Digital Image: Gray and RGB

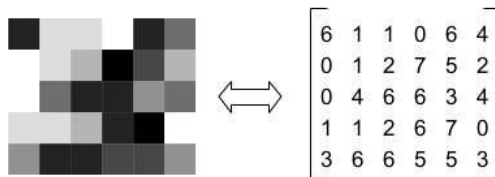
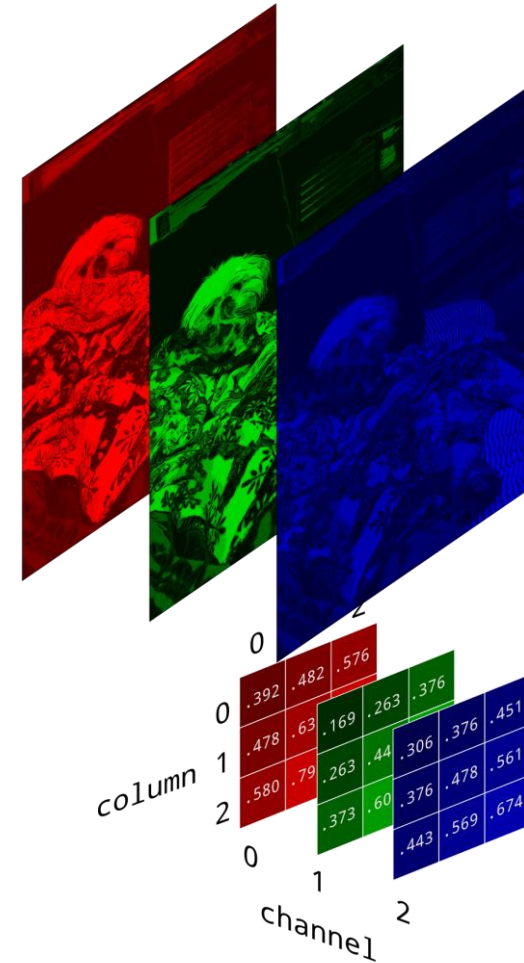


Fig. 8 Image credit: Diane Rohrer

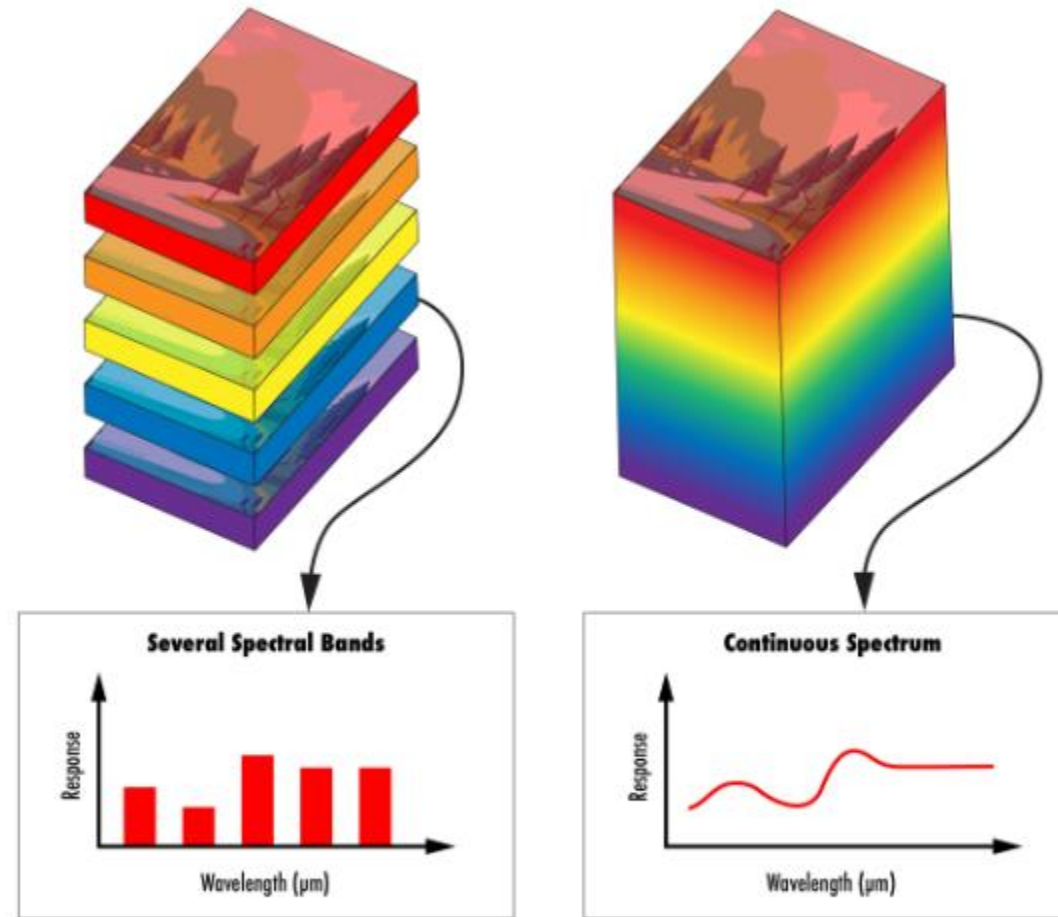
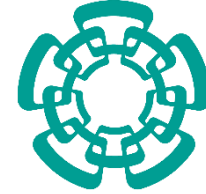
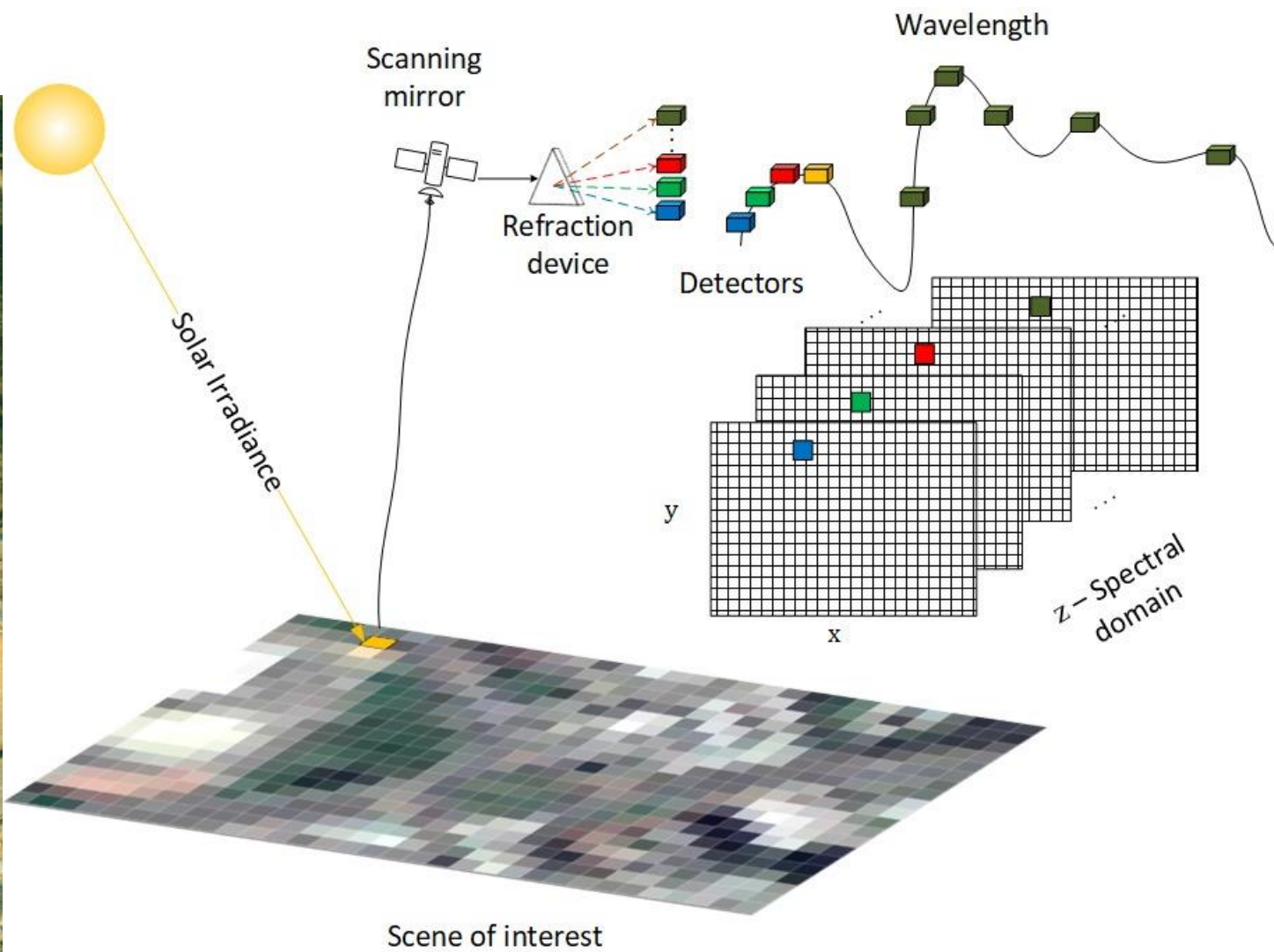


Figure 4: Comparison of the image stacks in multispectral imaging, in which there are images taken in several different spectra, and hyperspectral imaging, in which there are images taken in many different spectra.



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Phenomenology



Linear Systems (Inverse Problems)



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$$\mathbf{b} = \mathbf{A}\mathbf{x}$$

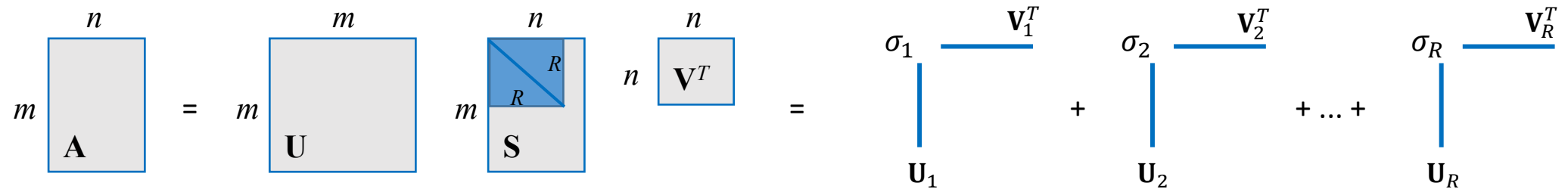
$$\mathbf{b} = \mathbf{A}\mathbf{x} + \mathbf{n}$$

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|$$

The Singular Value Decomposition (SVD)



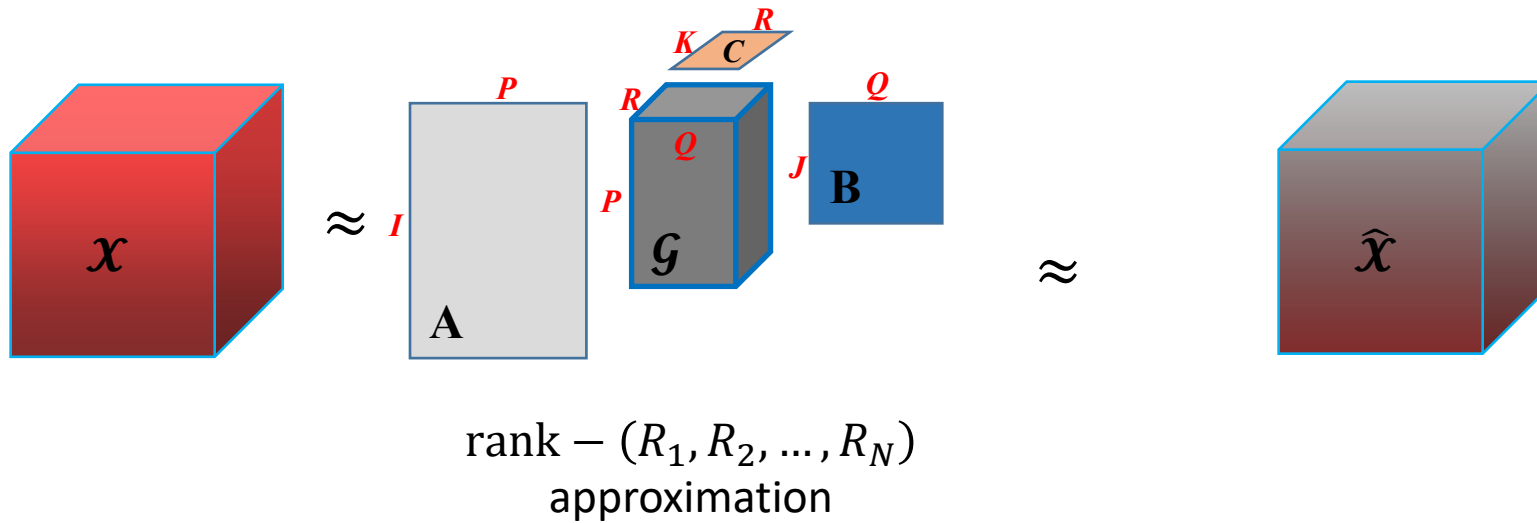
SVD



$$\mathbf{A} = \mathbf{USV}^T = \sum_{i=1}^R \sigma_i \mathbf{u}_i \mathbf{v}_i^T$$

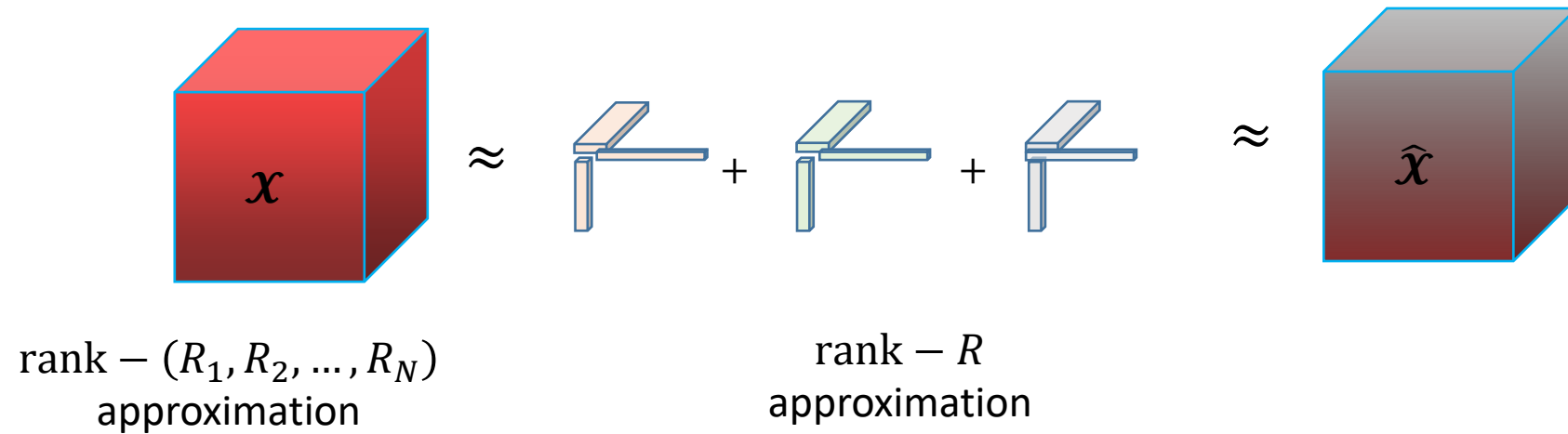
Tucker Decomposition

Dimensionality Reduction, Possible?



CP Decomposition

Dimensionality Reduction, Possible?

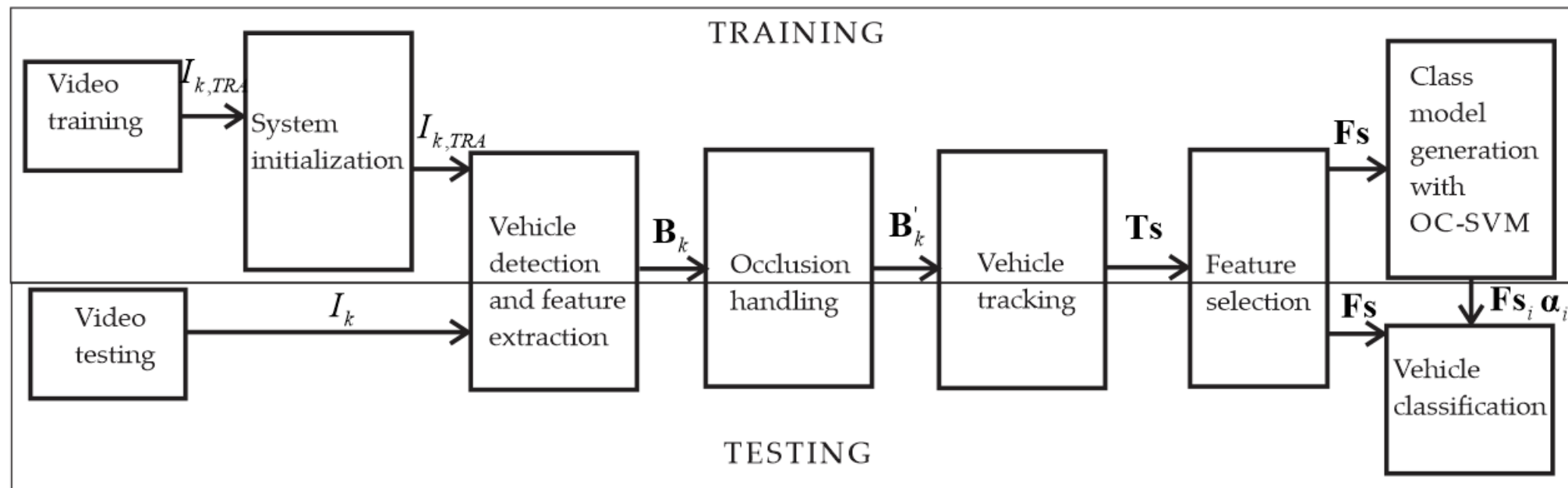


Content: Intelligent Transportation Systems (ITS) for an Internet of Things (IoT) Smart City



Moving objects are first segmented from the background using different techniques, e.g. the adaptive **Gaussian Mixture Model (GMM)**. After that, several **geometric features** are extracted, such as vehicle area, height, width, centroid, and bounding box. As occlusion is present, an algorithm was implemented to reduce it. The tracking is performed with adaptive Kalman filter. Finally, the **selected geometric features: estimated area, compactness, orientation, height, and width** are used by different classifiers to sort vehicles into several classes, e.g.,: **small, midsize, and large**.

State of the Art: Training and Testing





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Content: Our Data, Data need to be studied!

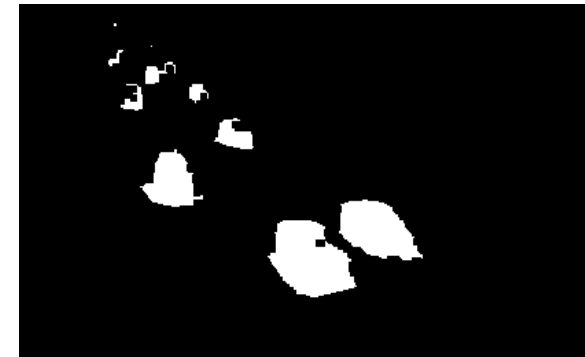
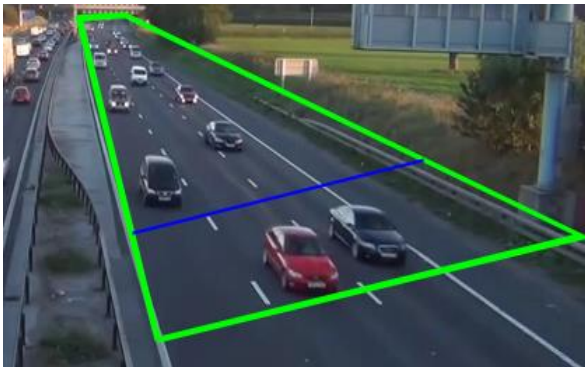
Experimental results in **8 real traffic videos** with more than **4000 ground truth vehicles** have shown that the improved system can run in real time under an **occlusion index of 0.312** and **classify vehicles** with a global **detection rate** or recall, precision, and other metrics up to **98.190 %**,

More than 10 typical metrics



Algorithms for Background and Foreground Formation

Challenge: Blobs, Probability Concepts, Entropy for Lane Detection



$$B_{x,y}^L = \operatorname{argmax} \left(H_{L_{x,y}}(n) \right)$$



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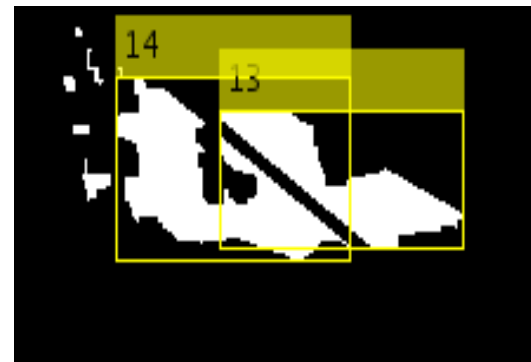
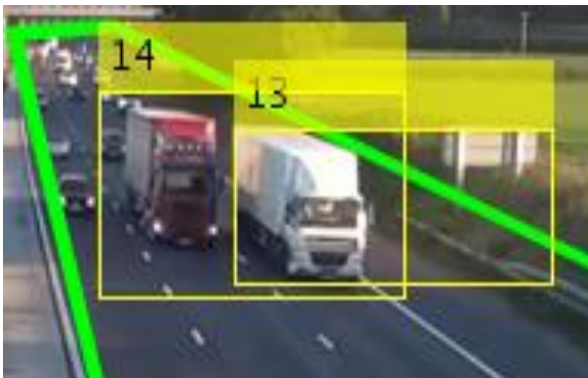
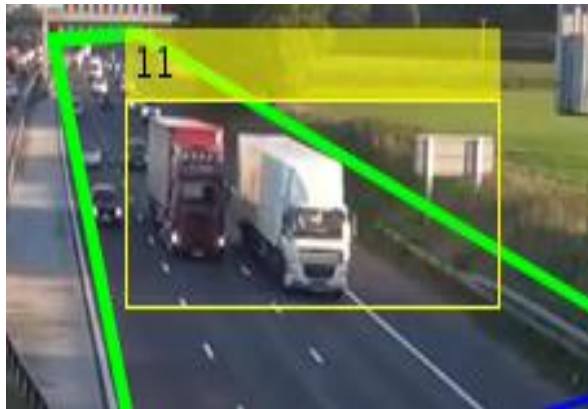
Challenge: Occlusion Handling

The challenge of any **occlusion-handling algorithm** in these scenarios is to minimize the effects of occlusion caused by **large vehicles** due to the high variance of their feature values, delivering a uniform space, which will be the input space for the classification stage



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Challenge: Occlusion Handling



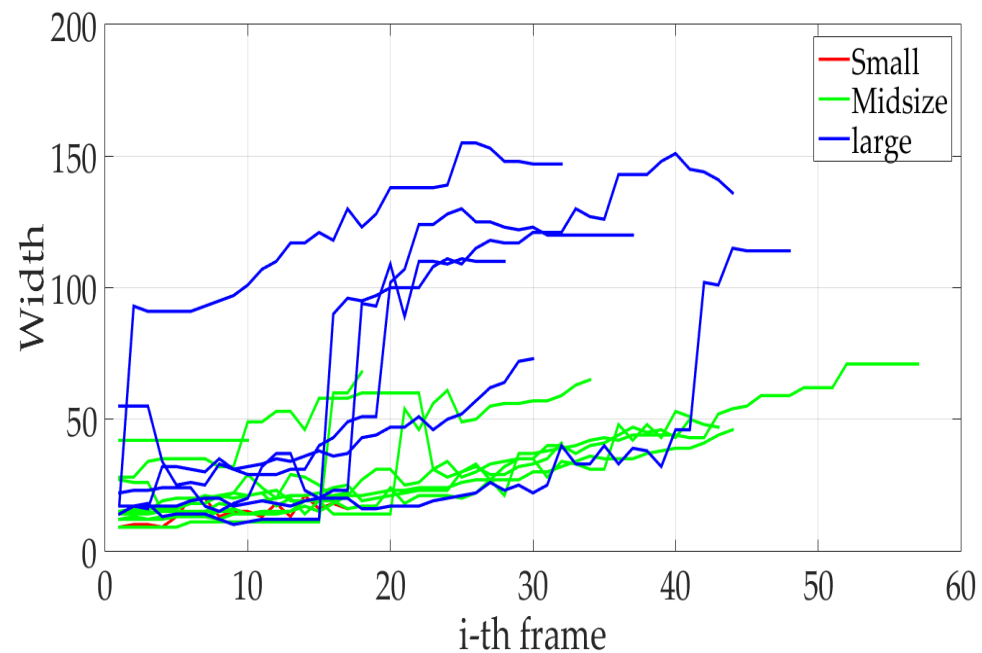
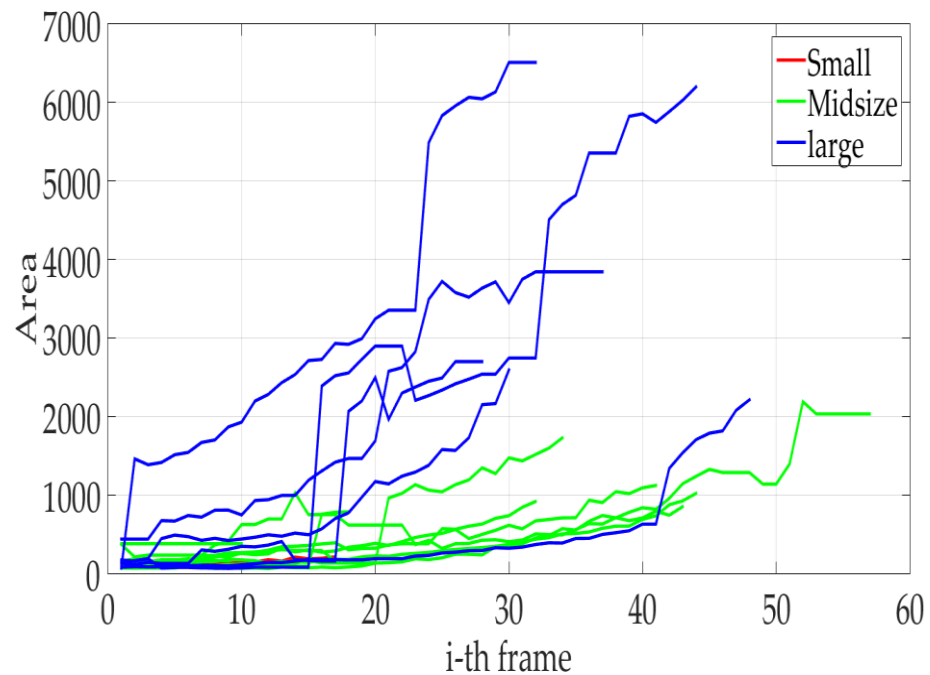


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Challenge: Features and Data, the Input Space

The challenge is to **find** and select **mathematically significant and/or invariant features** for a very **high performance** under different **weather** conditions and for several scenarios.

Challenge: Feature Behavior, Statistics



Challenge in the Tracking



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The challenge in the tracking is to find computational algorithms, that can locate any real moving object in a video with a low error.



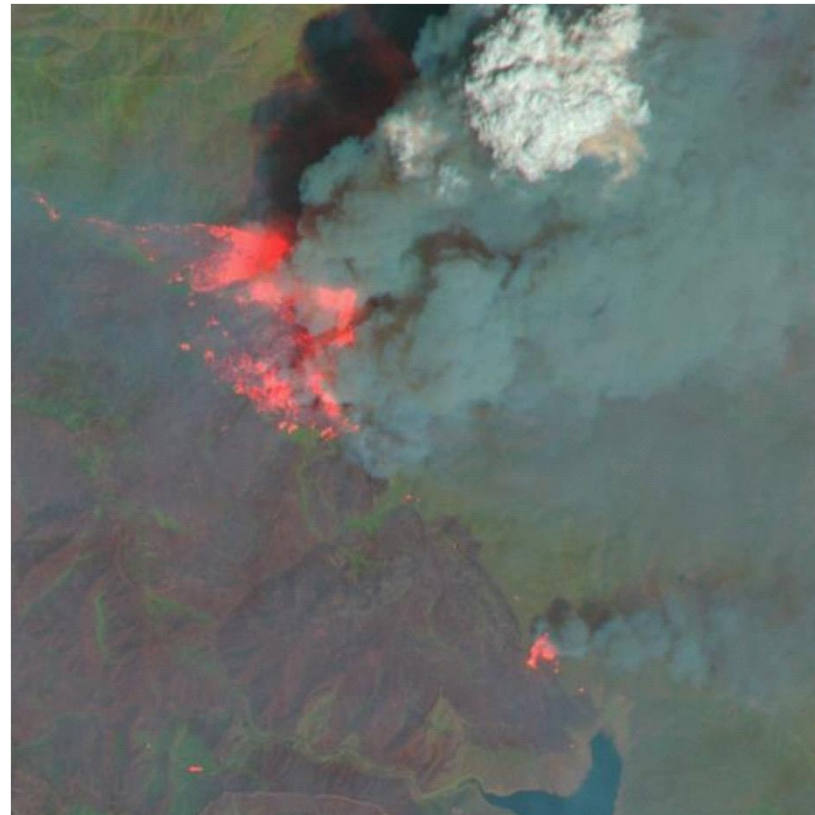
Challenge in the Classification

The challenge in the classification is to find **mathematical classifiers of the hypothesis set** that allow mapping every point of the input space to the corresponding classes of the output space with **minimal error**.

Challenge: Wildfire Detection at pixel level



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State of the Art: Mathematical Tools

- Probability and Statistic Methods
- Information Theory and Entropy
- Time Series Analysis
- Image and Image sequences: Vectors, Matrices and Tensors
- Linear and Multilinear Algebra
- Patterns and Clustering
- Machine Learning: Classification
- Several Algorithms

Results 3D feature Input Space (Very Goods)

Classification with OC-SVM and 3D feature input space

Input Space	TP	FP	FN	Recall	Precision	F-measure
10	7	14	3	70.000	33.333	45.161
2336	2298	6	38	98.373	99.739	99.051
141	137	25	4	97.163	84.567	90.429
2487	2442	45	45	98.190	98.190	98.190

Performance Metrics: Good and bad results



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$$\textit{Recall} = \frac{TP}{TP + FN}$$

$$\textit{Precision} = \frac{TP}{TP + FP}$$

Links to Videos: YouTubeTelecomCinvesGDL



- V1 <https://youtu.be/va0M-2-bobA>
- V2 <https://youtu.be/SeEhrogzXec>
- V3 https://youtu.be/BDGyB7XDV_E
- V4 <https://youtu.be/L8rEPmMO4x4>
- V5 <https://youtu.be/csdjk7hhtcE>
- V6 <https://youtu.be/FMLTJGfIOs1w>
- V7 <https://youtu.be/CBJ30IUo2x0>
- V8 <https://youtu.be/XGzb8VbpG2E>

Challenges

- Moving Object Detection
- Occlusion Handling, Vehicle Tracking, Vehicle Classification
- Lane Division Line Detection and Lane Formation
- Hyperspectral Images Processing for Dimensionality Reduction
- Semantic Segmentation with high performance metrics
- Image and Video Processing
- Hardware Implementation
- Wireless Communication
- Networking at different Levels

Mathematical Tools

- Features, measurements, observations: **scalars**
- Particular set of features: **vectors**
- Moving vehicles: **vectors(t)**
- Images, pixels: **Matrices, elements or entries**
- Multispectral and Hyperspectral images: **Tensors**
- Videos: **Tensors**
- Linear Algebra, Tensor Algebra
- Probability and Stochastic Processes: **Information Theory**



Some Publications of the Group

Content: Velazquez-Pupo, R.; Sierra-Romero, A.; Torres-Roman, D.; Shkvarko, Y.V.; Santiago-Paz, J.; Gómez-Gutiérrez, D.; Robles-Valdez, D.; Hermosillo-Reynoso, F.; Romero-Delgado, M. **Vehicle Detection with Occlusion Handling, Tracking, and OC-SVM Classification:** A High Performance Vision-Based System. *Sensors* 2018, *18*, 374.

Hermosillo-Reynoso, F.; Torres-Roman, D.; Santiago-Paz, J.; Ramirez-Pacheco, J. A Novel Algorithm Based on the **Pixel-Entropy for Automatic Detection of Number of Lanes**, Lane Centers, and Lane Division Lines Formation. *Entropy* 2018, *20*, 725.
<https://doi.org/10.3390/e20100725>

López, J.; Torres, D.; Santos, S.; Atzberger, C. Spectral Imagery **Tensor Decomposition** for Semantic Segmentation of Remote Sensing Data through Fully Convolutional Networks. *Remote Sens.* 2020, *12*, 517. <https://doi.org/10.3390/rs12030517>

G. D. Martín-del-Campo-Becerra, E. Torres-García, D. L. Torres-Román, S. A. Serafín-García and A. Reigber, "Statistical Regularization for TomoSAR Imaging With Multiple Polarimetric Observations," in *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 16, pp. 9539-9562, 2023, <https://doi.org/10.1109/JSTARS.2023.3310211>

González-Ramírez, A., Atzberger, C., Torres-Roman, D., López, J. (2025). **Representation Learning** of Multi-Spectral Earth Observation Time Series and Evaluation for Crop Type Classification. *MDPI/Remote Sensing*, *17*(3), 378.
<https://doi.org/10.3390/rs17030378>

E. Padilla-Zepeda, K. Alonso, R. D. L. Reyes, D. Torres-Roman, A. P. Pertiwi and T. Storch, "**Sentinel-2 Masking CNNs** Trained on Physics-Supervised Labels," in *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, <https://doi.org/10.1109/JSTARS.2025.3581058> pp 1-19, Date of Publication:19 June 2025

Hermosillo-Reynoso, F., Torres-Roman, D. (2024). A **Tensor Space for Multi-View and Multitask Learning** Based on **Einstein and Hadamard Products**: A Case Study on Vehicle Traffic Surveillance Systems. *Sensors*, *24*(23), 7463.
<https://doi.org/10.3390/s24237463>





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